TURING. *(TURING is amused.)* A few words of explanation?

KNOX. Yes.

TURING. In general terms?

KNOX. If possible.

TURING. It's about right and wrong. In general terms. It's a technical paper in mathematical logic, but it's also about the difficulty of telling right from wrong. *(brief pause)* People think — most people think — that in mathematics we always know what is right and what is wrong. Not so. Not any more. It's a problem that's occupied mathematicians for forty or fifty years. How can you tell right from wrong? Bertrand Russell wrote an immense book about it: *Principia Mathematica.* His idea was to break down all mathematical concepts and arguments into little bits and then show that they could be derived from pure logic; but it didn't quite work, all he was able to do was to show that it's terribly difficult to do anything of the kind. But it was an important book. Important and influential. It influenced both David Hilbert and Kurt Gödel. *(a brief digression)*

It's rather like what physicists call splitting the atom. As analyzing the physical atom has led to the discovery of a new kind of physics, so the attempt to analyze these mathematical atoms has led to a new kind of mathematics. *(resuming the main thread of his explanation)*

Hilbert took the whole thing a stage further. I don't suppose his name means much to you — if anything — well, there we are, that's the way of the world: people never seem to hear of the really great mathematicians. Hilbert looked at the problem from a completely different angle and he said, if we are going to have any fundamental system for mathematics — like the one Russell was trying to work out — it must satisfy three basic requirements: consistency, completeness and decidability. Consistency means that you won't ever get a contradiction in your own system; in other words, you'll never be able to follow the rules of your system and end up by showing that two and two make five. Completeness means that if any statement is true, there must be some way of proving it by using the rules of your system. And decidability means that there must exist some method, some definite procedure or test, which can be applied to any given mathematical assertion and which will decide whether or not that assertion is provable. Hilbert thought this was a very reasonable set of requirements to impose; but within a few years, Kurt Gödel showed that no system for mathematics could be both consistent and complete. He did this by constructing a mathematical assertion that said — in effect: "This assertion cannot be proved." A classic paradox. "This assertion cannot be proved." Well, either it can or it can't. If it can be proved, we have a contradiction, and the system is inconsistent. If it cannot be proved, then the assertion is true — but it can't be proved; which means that the system is complete. Thus mathematics is either inconsistent or it's incomplete. It's a beautiful theorem, quite beautiful. I think Gödel's theorem is the most beautiful thing I know. But the question of decidability was still unresolved. As I said, Hilbert thought there should be a single clearly defined method for deciding whether or not mathematical assertions were provable. The decision problem, he called it. The *Entscheidungsproblem*. In my paper "On

Computable Numbers," I wanted to show that there can be no one method that will work for all questions. Solving mathematical problems requires an infinite supply of new ideas. It was, of course, a monumental task to prove such a thing. One needed to examine the probability of all mathematical assertions past, present and future. How on earth could this be done? Eventually one word gave me the clue. People had been talking about the possibility of a mechanical process, a process that could be applied mechanically to solving mathematical problems without requiring any human intervention or ingenuity. Machine! — that was the crucial word. I conceived the idea of a machine, a Turing machine, that would be able to scan mathematical symbols — to read them, if you like — to read a mathematical assertion and to arrive at the verdict as to whether or not that assertion were provable. With this concept I was able to prove that Hilbert was wrong. My idea worked.

KNOX. You actually built this machine?

TURING. No, no — it was a machine of the imagination, like one of Einstein's thought experiments. Building it wasn't important; it's a perfectly clear idea, after all.

KNOX. Yes, I see; well, I don't, but I see something — I think. *(He looks at TURING.)* Forgive me for asking a crass and naive question — but what is the point of devising a machine that cannot be built in order to prove that there are certain mathematical statements that cannot be proved? Is there any practical value in all this?

TURING. The possibilities are boundless. In my paper, "On Computable Numbers," I explain how a special